1. Let $G$ be a graph such that $\chi(G - x - y) = \chi(G) - 2$ for all pairs $x, y$ of distinct vertices. Prove that $G$ is complete.

2. A graph is $k$-critical if $\chi(G) = k$ and $\chi(G - e) < k$ for any edge $e \in E(G)$. If $G$ is $k$-critical, show that $G$ does not contain a cut set consisting of pairwise adjacent vertices.

3. Show that every graph with chromatic number at least 4 contains a $K_4$-subdivision. (Hint: apply induction on the number of vertices)

4. Construct a graph $G$ with parallel edges (i.e. two vertices can have many edges between them), such that $\chi'(G) = 3\Delta(G)/2$.

5. Given a graph $G$, let $L(G)$ denote the graph where the vertices of $L(G)$ are the edges of $G$, and two vertices in $L(G)$ are adjacent if and only if the corresponding edges share a vertex in $G$. Prove that the number of edges in $L(G)$ is $\sum_{v \in V(G)} \left( \frac{d(v)}{2} \right)$.

6. Let $F$ be a family of $n$ unit squares with vertical and horizontal sides, such that no $k$ squares have a point in common. Prove that you can color the squares with $4k + 1$ colors, such that each color class consists of pairwise disjoint squares.