Graph theory - problem set 13
May 31, 2018

Exercises and problems

1. Find a minimum spanning tree in the following graph, once using the Tree-Growing Algorithm and once using the Forest-Growing Algorithm.

2. Prove that if a weighted graph has all weights distinct, then it has a unique minimum spanning tree.

3. Prove that the Forest-Growing Algorithm from the lecture correctly returns a minimum spanning tree.

4. Give a “pruning” algorithm that finds a minimum spanning tree by starting with the whole graph and removing edges while preserving connectedness. You do not have to prove that it works correctly, just convince yourself that it does.

5. Use Dijkstra’s algorithm to find a shortest path from a to b in the following two graphs.

6. Give polynomial-time algorithms for the following problems on a connected weighted undirected graph with positive weights. (You do not have to prove correctness or that it is really polynomial-time.)

   (a) Find the shortest cycle, if it exists.

   (b) Find the shortest path from a to b that has the fewest edges among all shortest ab-paths.

   (c) Find the widest path from a to b, i.e., the path for which the minimum weight of an edge is the largest.