1. Prove that a graph is bipartite if and only if it does not contain a cycle of odd length.

2. Prove the following asymmetric case of Hall’s theorem: If $G$ is a bipartite graph with bipartition $(A, B)$, such that, for any subset $X$ of $A$ we have $|N(X)| \geq |X|$, then there is a matching of size $|A|$ in $G$. We have denoted by $N(X)$ the neighborhood of $X$ in $B$, that is, the set of all vertices in $B$ adjacent to some vertex of $X$.

3. Prove that if $n$ is odd, $k = \lfloor n/2 \rfloor$, then there is a one-to-one correspondence between all $k$-element subsets and all $k+1$-element subsets of an $n$-element set such that every $k$-element set is contained in the corresponding $k+1$-element set.

4. Let $G$ is a bipartite graph with bipartition $(A, B)$, such that $|A| = |B| = n$. Prove that there exist a matching of size $t$ in $G$ if and only if for any subset $Y$ of $A$, we have $|N(Y)| \geq |Y| + t - n$, where we have denoted by $N(Y)$ the neighborhood of $Y$ in $B$.

5. A graph is called regular if the degrees of all vertices are equal. Prove that every regular bipartite graph with $2n$ vertices ($n$ in each part) of degree at least 1 has a matching of size $n$.

6*. Mr.Bean wants to type all the integer numbers from 1 to 999999 in ascending order. Since the keys 4 and 9 from his keyboard are missing, he does not type the numbers containing these digits. What will be the 2014-th numbered typed?