1. Which of the following relations, together with 
\[ \mathbb{Z}^2 = \{(x, y) : x, y \in \mathbb{Z}\} \]
form a partially ordered set:
   a. \((a, b) \preceq (c, d)\) if and only if \(a \leq c\) and \(b \leq d\).
   b. \((a, b) \preceq (c, d)\) if and only if \(a < c\) or \((a = c\) and \(b \leq d\)).
   c. \((a, c) \preceq (c, d)\) if and only if \(a \leq c\) or \(b \leq d\).

2. We say that \(a \preceq b\) if and only if \(a\) divides \(b\).
   a. Is \((\mathbb{N}, \preceq)\) a partially ordered set?
   b. Is \((\mathbb{Z}, \preceq)\) a partially ordered set?

3. You are given a group of women. Between two women \(A\) and \(B\), we define a relationship if \(A\) is the mother of \(B\). Does the set of women, together with this relationship becomes a partially ordered set? Justify your answer!

4. Prove that every sequence of \(mn + 1\) distinct numbers contains an increasing sequence of length \(n + 1\) or a decreasing sequence of length \(m + 1\).

5. Prove that for a bipartite graph \(G = (A \cup B, E)\), with all vertices in \(A\) having the same degree \(k\), and all vertices in \(B\) having the same degree \(l\), and \(k > l\), one can find a matching of \(A\).

6*. Suppose that 8 tennis players participated in a tournament, during which everybody played with all other players precisely one. Prove that it is possible to list the players in a single list so that each player (except the last one) won against the player who follows immediately after him/her on the list.