Discrete mathematics - problem set 6
October 23, 2014.

1. Prove that Kruskal’s algorithm works correctly also in the case when two or more edge weights are equal, and the algorithm picks one of the lowest weight edges randomly.

2. Apply Kruskal’s algorithm to the following graph to obtain a minimum spanning tree:

3. The following is called Prim’s algorithm:
   - *Initialize a tree with a single vertex, chosen arbitrarily from the graph.*
   - *Grow the tree by one edge: of the edges that connect the tree to vertices not yet in the tree, find the minimum-weight edge, and add it to the tree.*
   - *Repeat step 2 (until all vertices are in the tree).*

   Prove that, given a graph \( G \) as input, the output of Prim’s algorithm is a minimum spanning tree of \( G \).

4. Apply Prim’s algorithm to the graph from exercise 1.

5. Prove that a Kruskal-type algorithm for finding a minimum weight matching in a weighted graph would not work (select, at a time, an edge of minimum weight and add it to the matching).

6*. It is known that, in the northern hemisphere, the percentage of smokers among men is higher than the percentage of smokers among women. Also, in the southern hemisphere, the percentage of smokers among men is higher than the percentage of smokers among women. Is it true that globally, the percentage of smokers among men is higher than the percentage of smokers among women? Justify your answer!