1. In a house, each tenant uses exactly three rooms, and each room is used by exactly three tenants. Prove that the number of rooms equals to the number of tenants.

2. Prove that every graph with $n$ vertices and $n - 1$ edges that does not contain a cycle is a tree.

3. Prove that every tree with a vertex of degree $k$ has at least $k$ leaves.

4. In how many ways can you re-arrange the numbers 1, 2, ..., 10 such that no even number remains at its old position?

5. Prove that in every graph, there are two vertices having the same degree.

6. Prove that, among 6 people, either there are three who mutually know each other or there are three people who do not know each other (we assume that if $A$ knows $B$, then $B$ also knows $A$).

7*. An old lady has a blanket of unit area made up of five, possibly overlapping, patches. Each patch is of area at least $1/2$. Show that there are two patches whose overlap is of area at least $1/5$. 