Discrete mathematics - problem set 3

October 1, 2015.

1. What is the number of into (that is, injective) maps from a set of \( n \) elements to a set of \( m \) elements, where \( m \geq n \)?

2. There are \( n \) married couples attending a dance. How many ways are there to form \( n \) pairs for dancing if no wife should dance with her husband?

3. (a) Determine the number of permutations with exactly one fixed point.
   (b) Count the permutations with exactly \( k \) fixed points.

4. Which set of dominoes has larger cardinality:
   - dominoes containing numbers from 0 to 8 and admitting doubles (that means, any number can appear twice on the same domino piece) or
   - dominoes containing numbers from 0 to 9 without doubles (the two numbers appearing on the same domino piece must be distinct).

5. How many positive integers are there that divide \( 10^{40} \) or \( 20^{30} \)?

6. How many positive integers less or equal than 385 are there such that they are not divisible by neither of the following numbers: 5, 7, 11?

7. Prove that every tree with a vertex of degree \( n \) has at least \( n \) vertices of degree one.

8. Prove that \( \sum_{d|n} \phi(d) = n \) for every natural number \( n \), where the sum is taken over all natural numbers \( d \) dividing \( n \).

9 *. Can one place 28 points inside the cube of side length 3 in \( \mathbb{R}^3 \) such that the distance between any two points is at least 1.75? Justify your answer.