1. Let $C$ be the unit circle, and let $P_n$ denote an $n$-gon of minimum area circumscribed about $C$, where $n \geq 3$. Calculate the area of $P_n$.

2. Let $C$ be a convex disc in the plane, and let $p_n$ denote an $n$-gon of maximum area inscribed in $C$, $n \geq 3$. Similarly to the methods seen in class, prove that

$$A(p_n) \geq \frac{A(p_{n-1}) + A(p_{n+1})}{2}.$$ 

3. Let $C$ be a convex disc in the plane containing 0 and let the boundary of $C$ be parametrized as:

(a) $x(\phi) = r(\phi) \cos(\phi)$, $y(\phi) = r(\phi) \sin(\phi)$, where $r(\phi)$ is a non-negative periodic function with period $2\pi$, and $\phi$ runs from 0 to $2\pi$. Prove that

$$A(C) = \int_{0}^{2\pi} \frac{r(\phi)^2}{2} d\phi.$$ 

(b) Assume that $C$ has diameter 2, and $[-1, 1] \subset C$. Let the boundary be $x(\psi) = \cos(\psi)$, $y(\psi) = h(\psi) \sin(\psi)$, where $h(\psi) > 0$ is a periodic function of period $2\pi$. Show that

$$A(C) = \int_{0}^{2\pi} h(\psi) \sin^2 \psi d\psi.$$ 

4. For two points $p_1 = (x_1, y_1)$, $p_2 = (x_2, y_2)$ in the plane, we define their distance to be

$$d_1(p_1, p_2) = |x_1 - x_2| + |y_1 - y_2|,$$

this is called the $l_1$-metric, or taxicab (Manhattan) distance. Let $C$ be a circle of radius one. Show that for any set of 10 points $p_1, ..., p_{10}$ in the interior of $C$, there exist two indices $i, j \in \{1, ..., 10\}$, such that $d_1(p_i, p_j) < 4/3$.

5. Let $C$ be a convex disc in the plane, and let $T$ be the triangle of largest area inscribed in $C$. Show that in a suitable congruent copy of $2T$ contains $C$.

6. Determine the smallest area of a triangle circumscribed about the unit square $[0, 1]^2$. Show that the proportion between the areas of the smallest circumscribing triangle and the square is bigger than in the case of a circular disc and the smallest triangle circumscribed about it.

7*. Can you hang your diploma on the wall with a piece of string using two nails, so that if you remove any of the nails, the picture falls down?