1. In a kindergarten, there are 12 boys (3 of them are 3 years old, 5 are 4 years old and 4 are 5 years-old) and 9 girls (4 of them are 3 years-old, 2 are 4 years-old and 3 are 5 years old). Assuming that each child is picked with equal probability:
   - What is the probability of picking a girl?
   - What is the probability of picking a girl, provided that we pick a 3 years-old?
   - What is the probability of picking a 3 years-old, provided that it is a girl?
   The same question for boys.

2. Consider the set of all graphs on 4 labeled vertices and we pick one randomly from the set (each graph is picked with equal probability). What is the probability that the selected graph contains a triangle?

3. Let \( \{v_1, \ldots, v_n\} \) be unit vectors in \( \mathbb{R}^d \). Prove that it is possible to assign weights \( \epsilon_i \in \{\pm 1\} \) such that the vector \( \sum_{i=1}^n \epsilon_i v_i \) has Euclidean norm less than or equal to \( \sqrt{n} \).

4. In an \( n \times n \) matrix, each of the numbers 1, 2, \ldots, \( n \) appears exactly \( n \) times. Show that there is a row or a column in the matrix with at least \( \sqrt{n} \) distinct numbers.

5. Let \( \sigma \) be an arbitrary permutation of \( \{1, \ldots, n\} \), selected randomly from the set of all permutations. What is the expected value of the number of fixed points? Recall that \( i \) is a fixed point if \( \sigma(i) = i \).

6. Let \( \mathcal{F} \) be a family of 3-element subsets of a set \( X \). Prove that the elements of \( X \) can be colored with 3 colors so that at least \( |\mathcal{F}| \cdot 3! / 3^3 \) sets in \( \mathcal{F} \) have exactly one element of each color.

7*. In a school gym, there are 100 lockers numbered from 1 to 100. The first student who arrives opens all the lockers. The second changes the state of all the even-numbered lockers, the third one changes the state of all the lockers whose number is divisible by 3, and so on, until all the 100 students have passed (the students come one at a time). Which lockers are open after the 100-th student passes?