1. Prove that any tree on \( n \geq 2 \) vertices has at least 2 leaves (a leaf is a vertex of degree 1).

2. Show that in a tree containing an even number of edges, there is at least one vertex with even degree.

3. What is the Prüfer code of the following tree?

```
          3
          |
          6
          |
          1
          |
          10
          |
          4
        /   |
        7   8
        |
        2
        |
        9
        |
        5
```

4. Find the trees corresponding to the following Prüfer codes:
   
   \((1, 1, 2, 1, 4, 2)\); \((2, 4, 4, 3, 1, 1, 3, 2)\); \((4, 2, 1, 1, 3, 1, 4)\).

5. (a) Prove that if a vertex \( v \) of a tree \( T \) has degree \( d(v) \) then it appears exactly \( d(v) - 1 \) times in the Prüfer code of \( T \).
   
   (b) How many trees are there on the vertex set \([7]\) in which the vertices 2 and 3 have degree 3, the vertex 5 has degree 2 and all the others have degree 1?

6. (a) Describe which Prüfer codes correspond to stars (i.e. to trees where one vertex is connected to all other vertices).
   
   (b) Describe what trees correspond to Prüfer codes containing exactly 2 different values.
   
   (c) And which trees have all distinct values in their Prüfer codes?

7. Given a graph \( G \) and a vertex \( v \in V(G) \), \( G - v \) denotes the subgraph of \( G \) that we get by deleting the vertex \( v \) and all edges touching it from \( G \). Show that every connected graph \( G \) on at least two vertices contains some vertices \( x \) and \( y \) such that both \( G - x \) and \( G - y \) are connected.

8*. \( 2n \) men and \( 2n \) women sit around a round table. Prove that we can find \( 2n \) consecutive seats such that half of them are occupied by men and half of them by women.