1. Suppose $A_1, \ldots, A_m$ and $B_1, \ldots, B_m$ are subsets of $[n] = \{1, \ldots, n\}$ such that
   - $|A_i \cap B_i|$ is odd for every $1 \leq i \leq m$ and
   - $|A_i \cap B_j|$ is even for every $1 \leq i, j \leq m$ with $i \neq j$.
   Prove that $m \leq n$.

2. Let $A_1, \ldots, A_m$ be subsets of $[n]$ and let $s$ be a positive integer. Suppose that for every $i$, $|A_i|$ is not divisible by $s$, but the sizes of all pairwise intersections $|A_i \cap A_j|$ are divisible by $s$.
   (a) Prove that if $s = 7$ then $m \leq n$
   (b) Prove that if $s = 6$ then $m \leq 2n$.

3. Consider the reverse of the Oddtown problem: Suppose $A_1, \ldots, A_m$ are subsets of $[n]$ such that $|A_i|$ is even but $|A_i \cap A_j|$ is odd for every $1 \leq i \neq j \leq m$.
   (a) Show that $m \leq n + 1$.
   (b) Show that if $n$ is odd then $m \leq n$, and find sets $A_1, \ldots, A_n$ satisfying the conditions.

4. Let $v_1, \ldots, v_m$ be vectors in $\mathbb{R}^n$, all of whose coordinates are 0 or 1. Show that if they are linearly independent over the finite field $\mathbb{F}_p$ for some prime $p$, then they are also linearly independent over $\mathbb{R}$. 