1. Let \((a_n)\) be the sequence defined by \(a_0 = 1\) \(a_1 = 3\) and \(a_{n+2} = 2a_{n+1} - 2a_n\) for \(n \geq 0\). Solve the recurrence.

2. The purpose of this exercise is to guide you through another way of obtaining the explicit formula for the Fibonacci numbers.

   (a) Consider the generating function \(f(x) = \sum_{i=0}^{\infty} F_i x^i\) where \(F_0 = 0, F_1 = 1\) and \(F_n = F_{n-1} + F_{n-2}\), and show that \(xf(x) + x^2f(x) = f(x) - x\).

   (b) Deduce that \(f(x) = \frac{x}{1-x-x^2}\) (recall that every power series has a unique inverse, so dividing by \(1-x-x^2\) makes sense here).

   (c) Split \(f(x)\) into partial fractions by finding the roots \(x_{1,2}\) of \(1-x-x^2\) and the numbers \(A\) and \(B\) in \(\frac{x}{1-x-x^2} = \frac{A}{x-x_1} + \frac{B}{x-x_2}\).

   (d) Write \(\frac{A}{x-x_1} + \frac{B}{x-x_2}\) in its power series form and compare coefficients with \(f(x)\) to obtain the formulas.

3. Let \(a_n\) be the number of ways we can climb \(n\) stairs so that in each step we climb 1 or 3 stairs (so \(a_0 = a_1 = a_2 = 1, a_3 = 2\), etc.). Find a closed formula for the generating function \(\sum_{i=0}^{\infty} a_i x^i\).

4. Find a closed formula for the generating function of the sequence \((a_n)\) defined by \(a_0 = 0, a_1 = 1\) and \(a_{n+2} = 2a_{n+1} - a_n\) for \(n \geq 0\), and use it to obtain a closed formula for \(a_n\).

5. Compute the general term of the sequence \((a_n)\), where \(a_{n+2} = \sqrt{a_{n+1}a_n}\), \(a_0 = 2, a_1 = 8\), and find \(\lim_{n \to \infty} a_n\).