1. Suppose \( n \geq 2 \) and let \( H \) be an \( n \)-uniform hypergraph, i.e. each edge of \( H \) contains exactly \( n \) vertices. Prove that if \( H \) has fewer than \( 4^{n-1} \) edges, then there is a coloring of the vertices of \( H \) by 4 colors so that no edge is monochromatic.

2. Suppose \( n \geq 3 \) and let \( H \) be a family of \( n \)-element subsets of some set \( X \). Prove that if the number of sets in \( H \) is at most \( \frac{3^n}{2} - \frac{1}{2} \), then the elements of \( X \) can be colored by 3 colors so that every set in \( H \) contains all three colors.

3. Let \( A(x) \) be the generating function of a sequence \((a_0, a_1, a_2, \ldots)\). Using the power series \( A(x) \), write the generating functions of the following sequences:
   - (a) \((0, 0, \ldots, 0, a_0, a_1, \ldots)\).
   - (b) \((a_0, 0, \ldots, 0, a_1, 0, \ldots, 0, a_2, 0, \ldots, 0, \ldots)\).
   - (c) \((a_0, a_0, \ldots, a_0, a_1, a_1, \ldots, a_1, a_2, a_2, a_2, \ldots, a_2, \ldots)\).

4. Prove the following identities using generating functions.
   - (a) \( \binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \)
   - (b) \( \binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2 \)

5. (a) Prove that every power series \( \sum_{n=0}^\infty a_n x^n \) with \( a_0 \neq 0 \) has an inverse. The inverse of a power series \( A(x) = \sum_{n=0}^\infty a_n x^n \) is the power series \( B(x) = \sum_{n=0}^\infty b_n x^n \) such that \( A(x) \cdot B(x) = 1 \). We denote \( B(x) = 1/A(x) \).
   - (b) Find the inverses of the power series \( \sum_{n=0}^\infty x^n \), and \( \sum_{n=0}^\infty (n+1)x^n \).

6* Is it possible to color the positive rational numbers with two colors (red and blue) such that
   - both colors occur, and
   - if \( a, b \) have the same color then \( a + b \) has that color, as well?