Geometric Graph Theory

13. Exercise, 26. May, 2010
Wednesday 1015-1145*, MA A1 10

1. [Radon's Theorem] Prove that if we have $d+2$ points in $\mathbb{R}^d$, then they can be partitioned into two sets whose convex hulls intersect.
Hint: Use that $\sum x_i v_i = 0, \sum x_i = 0$ has a non-zero solution if $v_i \in \mathbb{R}^d$.

2. [Helly's Theorem, planar version] Prove that if from a finite collection of convex sets any three intersect, then they all have a common point.
Hint: First prove that any four intersect using the previous exercise.

3. Is it true that if from a planar collection of convex sets any three intersect, then they all have a common point?

4. Show that for any planar point set with n points, there is a point $p \in \mathbb{R}^2$ such that any line through this point has at most $2n/3$ points on both of its sides.

5. (*HW*) Show that there is a c such that for every n there is a planar graph that does not have a $c\sqrt{n}$ separator.

6. * Let $\mathcal{C}_0, \ldots, \mathcal{C}_d$ be a finite collection of convex sets in $\mathbb{R}^d$ such that for any choice of $C_i \in \mathcal{C}_i$ we have $\cap C_i \neq \emptyset$. Prove that there is an i such that $\cap \mathcal{C}_i \neq \emptyset$.

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoetoer.palvoelgyi@epfl.ch.