Geometric Graph Theory

12. Exercise, 18. May, 2010
Wednesday 1015-1145*, MA A1 10

1. (From last week.) Prove that between n points and n circles in the plane, the number of incidences is at most $O(n^{3/2})$.

2. Prove that the unit distance can occur $\Omega(n \log n)$ times among n points in the plane.
Hint: By induction get a recursion like $f(2n) \geq 2f(n) + n$.

Definition: If we have $2n$ points in the plane in general position, then we say that a segment connecting two of the points is a halving segment if its extension to a line cuts the points into two sets of size $n-1$. The graph whose vertices are the points and edges are the halving segments is denoted by G.

3. Show that if we reflect to a point the halving segments incident to it, then the halving segments and their reflections follow each other alternatingly.

4. Show that the degree of every point is odd in G.

5. (HW) Prove that $cr(G) + \sum_v \binom{(deg(v)+1)/2}{2} \leq \binom{n/2}{2}$.

6. * Let $f_k(n)$ be the largest number of edges that a graph with n vertices can have without having an edge that crosses k other edges.
a) Prove that $f_k(n) = O(n)$.
b) Prove that $f_2(n) \leq 4n - 8$ if $n \geq 3$.
c) Show that the inequality of b) is sharp for infinitely many n's.

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoetoer.palvoelgyi@epfl.ch.