Geometric Graph Theory

11. Exercise, 11. May, 2010
Wednesday 1015-1145*, MA A1 10

1. Prove using extremal graph theoretic methods (and not the crossing lemma) that there are at most
a) $O(n^{3/2})$ incidences (sum of the number of points for each line) between n points and n lines,
b) $mn^{1/2} + n$ incidences between n points and m lines. (By duality, also $m^{1/2}n + m$.)

2. Prove using the crossing lemma that between n points and m lines in the plane, the number of incidences is at most $O(n^{2/3}m^{2/3} + n + m)$. (This is known as the Szemerédi-Trotter theorem.)

3. Prove that between n points and n circles in the plane, the number of tangencies is at most $O(n^{3/2})$.

4. (*HW*) Show that if we have n points in the plane, then there are at most $O(n^2/k^3 + n/k)$ lines that contain at least k points.

5. * Prove that there is a K such that if we have n points in the plane, then either there is a line containing at least n/K points, or there are at least n^2/K lines containing at least two points.

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoetoer.palvoelgyi@epfl.ch.