Geometric Graph Theory

10. Exercise, 4. May, 2010
Wednesday 1015-1145*, MA A1 10

1. We have four random variables, X_1, X_2, X_3, X_4. We say that an event is *likely* if its chance of occurring is bigger than 50%. If $X_1 > X_2$ and $X_3 > X_4$ are both likely, then is $X_1 + X_3 > X_2 + X_4$ also necessarily likely?

2. We have $2n$ numbers, $x_1, y_1, \ldots, x_n, y_n$. We know that for all i $x_i > y_i$. Is it true that $\sum \alpha_i x_i > \sum \alpha_i y_i$ for any non-negative α_i's?

3. Consider the $n \times n$ square grid ($\{(i,j) : 0 \leq i < n, 0 \leq j < n\}$). Use the crossing lemma to show that the convex hull of any subset has at most $O(n^{2/3})$ vertices.
Hint: Shift the starting vertex of the lower-right part of the convex hull to every grid point and apply the crossing lemma to these n^2 curves.

4. Solve Exercise 3 using any other method.

5. (*HW*) Denote by *pair-cr(G)* the minimum number of pairs of edges that cross in a planar drawing of G. Prove that *pair-cr(G)* $\leq cr(G) \leq \binom{2\textit{pair-cr}(G)}{2}$.

6. * Suppose that in a multigraph, between any two vertices we have at most t edges. Prove that either $e = O(tn)$ or $cr(G) = \Omega(e^3/tn^2)$ where e is the number of edges.

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoetoer.palvoelgyi@epfl.ch.