1. We have four random variables, $X_1, X_2, X_3, X_4$. We say that an event is likely if its chance of occurring is bigger than 50%. If $X_1 > X_2$ and $X_3 > X_4$ are both likely, then is $X_1 + X_3 > X_2 + X_4$ also necessarily likely?

2. We have $2n$ numbers, $x_1, y_1, \ldots, x_n, y_n$. We know that for all $i$ $x_i > y_i$. Is it true that $\sum \alpha_i x_i > \sum \alpha_i y_i$ for any non-negative $\alpha_i$’s?

3. Consider the $n \times n$ square grid ($\{(i, j) : 0 \leq i < n, 0 \leq j < n\}$). Use the crossing lemma to show that the convex hull of any subset has at most $O(n^{2/3})$ vertices. Hint: Shift the starting vertex of the lower-right part of the convex hull to every grid point and apply the crossing lemma to these $n^2$ curves.

4. Solve Exercise 3 using any other method.

5. (HW) Denote by $\text{pair-cr}(G)$ the minimum number of pairs of edges that cross in a planar drawing of $G$. Prove that $\text{pair-cr}(G) \leq \text{cr}(G) \leq \binom{2 \text{pair-cr}(G)}{2}$.

6. * Suppose that in a multigraph, between any two vertices we have at most $t$ edges. Prove that either $e = O(tn)$ or $\text{cr}(G) = \Omega(e^3 / t n^2)$ where $e$ is the number of edges.

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoe-toer.palvoelgyi@epfl.ch.