Dilworth Theorem: In a finite poset, there exists an antichain, and a partition of the poset into a family of chains, such that the number of chains in the partition equals the cardinality of the antichain.

1. Show that any partial order can be extended to a complete order.

2. Using the Dilworth-theorem, prove Hall-theorem: In a bipartite graph $G$ the size of the biggest matching is the minimum number of vertices such that every edge contains at least one of them.

Definition: Define $\dim(P)$, the dimension of a partially ordered set to be the smallest $d$ such that there exists an injective mapping $f$ from $P$ to $\mathbb{R}^d$ for which $p < q$ if and only if every coordinate of $f(p)$ is smaller than the same coordinate of $f(q)$.

3. a) What can we say about the sets with $\dim(P) = 1$?
   b) Show that $\dim(P) \leq k$ if and only if $P$ is the intersection of $k$ total orderings.

4. Show that every partially ordered set has a dimension, i.e. that $\dim(P)$ really exists (if $P$ is finite). Moreover, show that it is at most $2n$ is $P$ has $n$ elements.

Definition: Define $G(P)$, the comparibility graph of a partially ordered set $P$ to be $V(G(P)) = P$ and $pq \in E(G(P))$ if and only if $p$ and $q$ are comparable, i.e. $p < q$ or $q < p$.

5. Prove that if $\dim(P) = 2$, then the complement of $G(P)$ is also a comparibility graph.

6. (HW) Suppose $P$ (finite) has a smallest element $m$, thus for any $p \in P$ we have $m \leq p$. Suppose that $f$ is a monotone function on $P$ meaning $x \leq y$ implies $f(x) \leq f(y)$. Show that $f$ has a fixed point meaning there is a $p$ for which $f(p) = p$.

7. * At most how much is the dimension of a partially ordered set on $n$ elements?

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoe-toer.palvoelgyi@epfl.ch.