Geometric Graph Theory

8. Exercise, 21. April, 2010
Wednesday 1015-1145*, MA A1 10

Dilworth Theorem: In a finite poset, there exists an antichain, and a partition of the poset into a family of chains, such that the number of chains in the partition equals the cardinality of the antichain.

1. Show that any partial order can be extended to a complete order.

2. Using the Dilworth-theorem, prove Hall-theorem: In a bipartite graph G the size of the biggest matching is the minimum number of vertices such that every edge contains at least one of them.

Definition: Define $dim(P)$, the dimension of a partially ordered set to be the smallest d such that there exists an injective mapping f from P to $\mathbb{R}^d$ for which $p < q$ if and only if every coordinate of $f(p)$ is smaller than the same coordinate of $f(q)$.

3. a) What can we say about the sets with $dim(P) = 1$?
b) Show that $dim(P) \leq k$ if and only if P is the intersection of k total orderings.

4. Show that every partially ordered set has a dimension, i.e. that $dim(P)$ really exists (if P is finite). Moreover, show that it is at most $2n$ is P has n elements.

Definition: Define $G(P)$, the comparibility graph of a partially ordered set P to be $V(G(P)) = P$ and $pq \in E(G(P))$ if and only if p and q are comparable, i.e. $p < q$ or $q < p$.

5. Prove that if $dim(P) = 2$, then the complement of $G(P)$ is also a comparibility graph.

6. (*HW*) Suppose P (finite) has a smallest element m, thus for any $p \in P$ we have $m \leq p$. Suppose that f is a monotone function on P meaning $x \leq y$ implies $f(x) \leq f(y)$. Show that f has a fixed point meaning there is a p for which $f(p) = p$.

7. * At most how much is the dimension of a partially ordered set on n elements?

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoetoer.palvoelgyi@epfl.ch.