Geometric Graph Theory

Wednesday 1015-1145*, MA A1 10

0. There are four military vessels in the Persian Gulf. The captains of the ship are exactly 100 yards from each other. The first is a battleship, the second is a cruiser, the third is a carrier. What is the fourth?

1. Show that $G$ is outerplanar if and only if $G'$, that we obtain by adding one more vertex to $G$ that is connected to all its vertices, is planar.

2. Suppose that we have a baguette with cheese. Prove that we can always cut it into two such that both of us gets the same amount of baguette and cheese.
A more mathematical version is to prove that if we have $2n$ red and $2n$ blue points in the plane, no three on a line, then there is a line having $n - n$ points of both colors on both of its sides.

3. Prove that the unit distance can occur at most $n^{3/2} + O(n)$ times among $n$ points in the plane. (Now it is not the maximal distance!)

4. (HW) Prove that if a connected graph contains no cycles or $Y$’s, then it is a caterpillar. ($Y$ is the tree on seven vertices where the central vertex has degree three, its three neighbors have degree two and their neighbors are the leafs. A caterpillar is such a tree, that deleting its leafs we get a path.)

5. * Suppose that we have a baguette with ham and cheese. Prove that we can always cut it into two such that both of us gets the same amount of baguette, ham and cheese.
A more mathematical version is to prove that if we have $2n$ red, $2n$ blue and $2n$ green points in the space, no four on a plane, then there is a plane having $n - n$ points of all three colors on both of its sides.

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoe-toer.palvoelgyi@epfl.ch.