Geometric Graph Theory

5. Exercise, 24. March, 2010
Wednesday 1015-1145*, MA A1 10

1. Suppose that the biggest distance among n points in the plane is 1. Prove by induction that it can occur at most n times.

2. Suppose we have n points in the plane.
a) Show that there is a non-selfintersecting Hamiltonian-cycle connecting them whose edges are straight-line segments. (Ie. connect all the points with segments such that you obtain a cycle with no intersections.)
b) Prove that the shortest Hamiltonian-cycle is non-selfintersecting.

3. a) Show that a convex geometric graph with no k pairwise disjoint edges can have $(k-1)n - C_k$ edges.
b) What if all degrees are at most $2k$?

4. Show that a geometric graph with no 3 pairwise disjoint edges can have $2.5n - 3$ edges for arbitrarily large n.

Definition: We say that a graph is outerplanar if it has a non-crossing planar drawing (with NOT necessary straight-line segments) such that all vertices are on the outer face, ie. the outer face has $|V(G)|$ vertices.

5. (*HW*) Analyze outerplanar graphs. At most how many edges can an outerplanar graph with n vertices have? Prove that the chromatic number of outerplanar graphs is at most three.

6. * Prove that a graph is outerplanar if and only if it does not contain a subdivision of a K_4 or a $K_{2,3}$.

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoetoer.palvoelgyi@epfl.ch.