1. Argue that the star with $n + 1$ vertices (K$_{1,n}$) can be embedded into an $O(\sqrt{n}) \times O(\sqrt{n})$ size grid by approximating the number of relative prime pairs and proving that $\prod_{p \text{ prime}} (1 - \frac{1}{p^2})$ is finite.

2. What is the crossing number of
   a) $K_{3,3}$?
   b) $K_{3,4}$?
   c) $K_{4,4}$?
   d) $K_{3,n}$?

3. Prove that $\lim \frac{cr(K_{n,n})}{n^4}$ exists and it is positive.

4. (HW) What is the biggest $n$ such that $K_n$ can be embedded into the projective plane without crossings? (The projective plane is the easiest to represent as a disc and if an edge exits, then it returns on the opposite side. You can use the respective Euler formula without proving it.)

5. * Drawing $K_4$ shows that any graph on 4 vertices can be embedded into a $2 \times 2$ grid, so for $n = 4$ it is not necessary to use a $(2n - 4) \times (n - 2)$ grid, we can represent any graph on four vertices on a smaller grid. Can you prove a good lower bound, ie. construct a graph that needs a large grid (approximately $n^2$) to be drawn into?

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoteor.palvoelgyi@epfl.ch.