Geometric Graph Theory

3. Exercise, 10. March, 2010
Wednesday 1015-1145*, MA A1 10

1. Argue that the star with $n+1$ vertices ($K_{1,n}$) can be embedded into an $O(\sqrt{n}) \times O(\sqrt{n})$ size grid by approximating the number of relative prime pairs and proving that $\prod_{p\ prime}^{\infty}\left(1-\frac{1}{p^2}\right)$ is finite.

2. What is the crossing number of
a) $K_{3,3}$?
b) $K_{3,4}$?
c) $K_{4,4}$?
d) $K_{3,n}$?

3. Prove that $\lim cr(K_{n,n})/n^4$ exists and it is positive.

4. (*HW*) What is the biggest n such that K_n can be embedded into the projective plane without crossings? (The projective plane is the easiest to represent as a disc and if an edge exits, then it returns on the opposite side. You can use the respective Euler formula without proving it.)

5. * Drawing K_4 shows that any graph on 4 vertices can be embedded into a 2×2 grid, so for $n = 4$ it is not necessary to use a $(2n-4) \times (n-2)$ grid, we can represent any graph on four vertices on a smaller grid. Can you prove a good lower bound, ie. construct a graph that needs a large grid (approximately n^2) to be drawn into?

New exercises and notes can be found at http://dcg.epfl.ch/page85509.html
Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoetoer.palvoelgyi@epfl.ch.