1. Prove that for any connected graph, \( G \), we have \( e(G) \geq v(G) - 1 \) with equality if and only if \( G \) contains no cycles. Such graphs are called trees.

2. Prove that any planar graph can be triangulated, i.e., extended to a triangulation by adding new non-crossing edges to it.

![Petersen graph](image)

Figure 1: Petersen graph.

3. a) Show that the Petersen graph (on Figure) is not planar.
b) How many edges do we need to delete from it to make it planar?
c) Does it matter which two?

4. Let \( G \) be a planar graph, whose exterior face is a cycle.
a) Prove that there are at least two vertices of this cycle that are connected to only two other vertices from the cycle.
b) Can this be improved to three, if the cycle is large enough?

5. \((HW)\) Let \( \{p_1, \ldots, p_n\} \) be a set of \( n \geq 3 \) different points in the plane such that the smallest distance between them is 1 cm.
a) Show that the number of pairs which are at distance 1 from each other, is at most \( 3n - 6 \).
b) Is this bound tight? Why?
c) Show that for any large enough \( n \), there is a configuration with the smallest distance appearing at least \( 2.99n \) times.

6. * Show that the chromatic number of planar graphs is at most five.

New exercises and notes can be found at [http://dcg.epfl.ch/page85509.html](http://dcg.epfl.ch/page85509.html)

Solutions to selected homeworks should be handed in at the beginning of the next session or sent to doemoetoer.palvoelgyi@epfl.ch.