1. Let us assume that we have \( n \) intervals of \( \mathbb{R} \) whose endpoints are different. Prove that if any two intersect, then all of their intersection is non-empty.

**Definition:** Define \( \text{dim}(P) \), the dimension of a partially ordered set to be the smallest \( d \) such that there exists an injective mapping \( f \) from \( P \) to \( \mathbb{R}^d \) for which \( p < q \) if and only if every coordinate of \( f(p) \) is smaller than the same coordinate of \( f(q) \).

2. a) What can we say about the sets with \( \text{dim}(P) = 1 \)?
   b) Show that \( \text{dim}(P) \leq k \) if and only if \( P \) is the intersection of \( k \) total orderings.

**Definition:** Define \( G(P) \), the comparability graph of a partially ordered set \( P \) to be \( V(G(P)) = P \) and \( pq \in E(G(P)) \) if and only if \( p \) and \( q \) are comparable, i.e., \( p < q \) or \( q < p \).

3. Prove that if \( \text{dim}(P) = 2 \), then the complement of \( G(P) \) is also a comparability graph.

4. Let us assume that we have \( n \) intervals of \( \mathbb{R} \) whose endpoints are different. Prove that if no \( k + 1 \) intervals have a point in common, then one can color the intervals with \( k \) colors so that no two intervals from the same color class intersect.

5. Let us assume that we have \( n^2 \) intervals of \( \mathbb{R} \) whose endpoints are different. Prove that there are \( n \) intervals that are all disjoint or there is a point contained in \( n \) intervals.

6. All partially ordered sets are representable by \( \mathbb{R} \to \mathbb{R} \) functions \((f < g \text{ if } \forall x f(x) < g(x))\).

7. (HW) Finish the proof of Dilworth’s theorem that starts with deleting the max and min element.

8. * Prove that in a complete geometric graph, there are always \( c\sqrt{n} \) pairwise crossing edges.

New exercises and notes can be found at http://dcg.epfl.ch/page78315.html
Solutions to selected homeworks should be sent to doemoetoer.palvoelgyi@epfl.ch or handed in at the beginning of the next session.