1. Which of the following sets contains only non-planar graphs?
   a) \( \{K_{2,4}, K_6\} \)
   b) the set of graphs with 2015 vertices and 6035 edges
   c) the set of bipartite graphs with 2015 vertices and 4035 edges
   d) the set of graphs with minimum degree 5

2. Which of the following sets contains at least one non-planar graph?
   a) the set of graphs that contain no subdivision of \( K_5 \) or \( K_{3,3} \)
   b) the set of graphs that do not contain \( K_5 \) and \( K_{3,3} \) as a minor
   c) the set of graphs that contain no subdivision of \( K_5 \) and do not contain \( K_{3,3} \) as a minor
   d) none of the above

3. The maximum possible chromatic number of a bipartite planar graph is
   a) 2
   b) 3
   c) 4
   d) not defined, because it can be arbitrarily large

4. The crossing number of \( K_5 \) is
   a) 0
   b) 1
   c) 5
   d) 10

5. Let \( D \) be a drawing of a graph \( G \). Which of the following conditions guarantees that \( D \) is planar?
   a) every pair of adjacent edges in \( D \) cross an even number of times
   b) every pair of independent edges in \( D \) cross an even number of times
   c) every pair of edges in \( D \) cross an odd number of times
   d) every pair of edges in \( D \) cross at most once

6. Which statement is true?
   a) Every planar graph with \( n \) vertices and \( n \) edges can be drawn as a thrackle.
   b) Every bipartite planar graph can be drawn as a thrackle.
   c) Every bipartite planar graph can be drawn as an odd thrackle.
   d) none of the above
7. Let $M = \{5, 7, 9, 11, 13, 15, \ldots, 2015\}$. Which of the following is a partially ordered set?

a) $(M, \preceq_a)$ where $m \preceq_a n$ means that $m$ divides $n$

b) $(M, \preceq_b)$ where $m \preceq_b n$ means that $m \leq n$ and there is a prime that divides both $m$ and $n$

c) $(M \times M, \preceq_c)$ where $(m_1, m_2) \preceq_c (n_1, n_2) \iff (m_1 \leq n_1 \text{ or } m_2 \leq n_2)$

d) $(M \times M, \preceq_d)$ where $(m_1, m_2) \preceq_d (n_1, n_2) \iff m_1 + m_2 \leq n_1 + n_2$

8. Let $P = (2^5, \subseteq)$ be a poset of subsets of $[5] = \{1, 2, 3, 4, 5\}$ ordered by inclusion. Which of the following subsets is an antichain in $P$?

a) all subsets of $[5]$ that contain 1

b) all subsets of $[5]$ that do not contain 1

c) all subsets of $[5]$ that have exactly three elements

d) none of the above

9. For a graph $G$, let $\omega(G)$ be the number of vertices of the largest complete subgraph of $G$, and let $\alpha(G)$ be the size of the maximum independent set of $G$. Which statement is true for every comparability graph $G$ with $n$ vertices?

a) $\omega(G) \cdot \alpha(G) \leq n$

b) $\omega(G) \cdot \alpha(G) \geq n$

c) $G$ is bipartite

d) none of the above

10. Which statement is true?

a) Every planar graph can be represented as the touching graph of non-overlapping discs in the plane, each of which has radius at least 1 and at most 10.

b) The vertex set of every planar graph with $n$ vertices can be partitioned into four sets $A, B, C, S$ such that there are no edges between $A, B$ and $C$, and the sizes of the sets satisfy $|A|, |B|, |C| \leq 9n/10$ and $|S| \leq 10n^{1/3}$.

c) Every planar graph with $n$ vertices has a straight-line drawing in the plane without crossings such that the longest edge is at most $10n$ times longer than the shortest edge.

d) none of the above

11. Suppose that both $G$ and its complement $G^\complement$ are planar graphs. Prove that $G$ has at most ten vertices.

12. Prove that if $D$ is a drawing of $G$ with minimum possible number of crossings, then adjacent edges do not cross in $D$.


14. Prove that if $G$ is a geometric graph with no two disjoint edges, then $|E(G)| \leq |V(G)|$.

15. Prove that every graph $G$ with $m$ edges has a bipartite subgraph with at least $m/2$ edges.