1. Prove that for \( p(n) \ll \ln n/n \), \( G(n, p(n)) \) will almost always contain at least one isolated vertex.

2. Prove that the threshold function for the existence of cycles of length 4 in \( G(n, p(n)) \) is \( p(n) = 1/n \).

3. Recall that the random bipartite graph \( BG(n, p) \) is defined as a probability space over the set of graphs on the vertex set \( A \cup B \) with \( |A| = |B| = n \), and for each \( a \in A, b \in B \), the pair \( ab \) forms an edge independently with probability \( p \). A perfect matching in the graph is a set of \( n \) pairwise vertex-disjoint edges.

   (i) Compute the expected number of perfect matchings in \( BG(n, p) \).

   (ii) Give an asymptotic estimate for the value \( p(n) \) for which the expected number of perfect matchings is 1.

4. There are 33 rooks placed on an 8 \( \times \) 8 chessboard. Prove that one can find 5 of them such that any two lie in different rows and different columns.