Suppose $X$ is a random variable which is the sum of independent random variables with $\mathbb{E}(X) = \mu$ and let $\epsilon > 0$. The Chernoff’s bound states that there exists a constant $c_\epsilon > 0$ such that

$$\Pr(|X - \mu| > \epsilon \mu) < 2e^{-c_\epsilon \mu}.$$  

Furthermore, we have

$$c_\epsilon = \min\{\frac{\epsilon^2}{2}, (1 + \epsilon) \ln(1 + \epsilon) - \epsilon\}.$$  

1. For any $\epsilon > 0$ and $p(n) \gg \frac{\ln n}{n}$, prove that the degree of every vertex in $G(n, p(n))$ will almost always lie between $(1 - \epsilon)(n - 1)p(n)$ and $(1 + \epsilon)(n - 1)p(n)$.

*Hint: Use Chernoff’s bound stated above!*  

2. Let $S$ be a finite set of points in the plane. Prove that there exists a 2-coloring $c : S \to \{-1, +1\}$ so that on every horizontal and vertical line in the plane, the sum of values of $c$ is $+1, -1$ or 0.

3. Use Chernoff’s bound to show that the probability that every pair of distinct vertices of $G(n, 1/2)$ have a common neighbor tends to 1 as $n \to \infty$.  