1. Let \( X_1, X_2, \ldots, X_n \) be independent random variables taking values \(-1, +1\) such that for all \( i \), we have
\[
\Pr(X_i = 1) = \Pr(X_i = -1) = \frac{1}{2}.
\]
Denote \( X = \sum_{k=1}^{n} X_k \). Compute the following expected values:
(i) \( \mathbb{E}(X^2) \).
(ii) \( \mathbb{E}(X^4) \).

2. Let \( A = (a_{ij})_{i,j=1}^{n} \) be a random \( n \times n \) \( \{0, 1\} \)-matrix such that all the entries are independent and for all \( i, j \) we have
\[
\Pr(a_{ij} = 0) = \Pr(a_{ij} = 1) = \frac{1}{2}.
\]
We define the **Permanent** of \( A \) as
\[
\text{Per}(A) = \sum_{\pi} \prod_{k=1}^{n} a_{k\pi(k)},
\]
where the sum is taken over all the permutations of \( \{1, 2, 3, \ldots, n\} \). Compute \( \mathbb{E}(\text{Per}(A)) \).

3. A **Tournament** \( T = (V_T, E_T) \) is a complete directed graph on the vertex set \( V_T \), i.e. for all distinct vertices \( v, v' \in V_T \), exactly one of \((v, v'), (v', v)\) belongs to \( E_T \). Prove that every tournament \( T \) has a path which visits each vertex of \( T \) exactly once. (Such a path is called a **Hamiltonian path** in \( T \).)

4. Let \( K_n \) denote a complete graph with \( n \) vertices. Given any positive integers \( k \) and \( l \), the **Ramsey number** \( R(k, l) \) is defined as the smallest integer \( n \) such that in any two-colouring of the edges of \( K_n \) by red and blue, either there is a red \( K_k \) or a blue \( K_l \). Prove that \( R(k, l) \leq \binom{k+l-2}{k-1} \).

5. An \( n \times n \) chess board is broken so that only the diagonal and the upper and lower levels are left. See Figure 1 for \( n = 10 \). We start from the lower left corner square \( A \) and at each step, we can go to the right or upper adjacent squares, so in the first step, we are allowed to move to one of the squares \( C \) or \( D \). Compute the number of ways we can reach the upper right corner square \( B \).

![Figure 1](image-url)
§ 6. Consider the lower triangular part of an $n \times n$ chess board. See Figure 2 for $n = 10$. Compute the number of ways one can start from the square $A$ and reach the square $B$ with the same conditions as in Problem 5.

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§ — an optional contest problem. You may submit a solution until the beginning of the next lecture.