1. What is the crossing number of
   a) $K_5$?
   b) $K_6$?

2. What is the crossing number of
   a) $K_{3,3}$?
   b) $K_{3,n}$? (Hint: use induction.)

3. Let $D$ be a drawing of $K_n$ in the plane, with possible edge crossings. Show that if every two edges have at most one point in common (either an endpoint or a crossing), then there are at most $\binom{n}{4}$ crossings in $D$.

4. Prove that $\lim_{n\to\infty} cr(K_{n,n})/(\binom{n}{2})^2$ exists and it is positive.

5. Place $2n$ red points and $2n$ blue points on the unit sphere (the surface of a ball with radius 1), so that every point is directly opposite to a point of the same color, but no great circle contains more than four points (a great circle is a circle in the unit sphere with radius 1). Connect every red point with every blue point by the shortest arc on the sphere. Show that the resulting drawing of $K_{2n,2n}$ has exactly $n^2(n-1)^2$ crossings.

6. Find a drawing of the graph $K_5$ in the plane such that every two edges have at most one point in common (either an endpoint or a crossing), the total number of crossings is 5, and there is no cycle of length 5 without crossings.

— an optional homework. You may submit a written solution of this problem until the beginning of the next lecture and receive our feedback.

— an optional contest problem. You may submit a solution until the beginning of the next lecture. We encourage you to participate.

New exercises and notes can be found at http://dcg.epfl.ch/page-117408-en.html.