1. Show that subsets of Hamilton paths form an intersection of 3 matroids.

More precisely, given a directed graph \( D \) and \( s, t \in V(D) \), let

\[
\mathcal{I} = \{ Q \subseteq E(D) : \exists \text{ Hamilton path } P \text{ from } s \text{ to } t \text{ such that } Q \subseteq P \}.
\]

Show that \((E(D), \mathcal{I})\) is an intersection of 3 matroids.

2. Given an undirected graph \( G = (V, E) \), an orientation is a directed graph \( D = (V, E') \) with a bijection \( \varphi : E' \to E \) such that \( \varphi(ab) = \{a, b\} \). In other words, each edge \( \{a, b\} \in E \) is given a direction, either \( ab \) or \( ba \).

Given \( k : V \to \mathbb{N} \), show that the problem of finding an orientation such that

\[
\delta^{\text{in}}(v) = k(v)
\]

for each \( v \in V \), or showing that none exists, can be solved using the matroid intersection algorithm.

3. Use the matroid intersection algorithm to show that there is no simultaneous spanning tree in the following two graphs (i.e., there is no \( T \subseteq \{a, b, \ldots, j\} \) that is a spanning tree in both).

4. Make up 2 matroids such that the matroid intersection algorithm needs at least 2 non-greedy steps (i.e. with \(|Q| > 1\)) to get a maximum common independent set.