General Matchings

1. Consider the following greedy algorithm for matchings:
   \[ \text{Repeatedly add } uv \text{ such that } \deg(u) + \deg(v) \text{ is minimal, then remove } \delta(u) \cup \delta(v). \]
   Give an example for which this algorithm does not return a maximum cardinality matching.

2. Execute the blossom algorithm to find a maximum cardinality matching for the following example, starting with the given matching.

3. Execute the postman algorithm to find a minimum postman tour in the following example (find the minimum weight perfect matching by inspection).

4. Show that Dijkstra’s algorithm can be used to find a shortest \( st \)-path in an undirected weighted graph, if all weights are nonnegative. Why does this fail when there are negative weights?
   Give an algorithm for shortest \( st \)-paths in an undirected graph \( G \) with arbitrary weights, but without negative cycles.