You can hand in one of the following problems at the start of Tuesday’s problem session. Please explain your solution carefully. Don’t forget to put your name.

Spanning Trees

1. Prove carefully that the Forest-Growing Algorithm given in class returns a minimum spanning tree, if it exists.

2. Give a greedy algorithm that finds a minimum spanning tree by removing edges while preserving connectedness. Prove that it works. Give an example to show that the same approach does not work for minimum spanning directed trees.

3. Prove that if its weight function $w$ is injective, then a graph has at most one minimum spanning tree.

4. Give an algorithm that finds the second-smallest spanning tree in a weighted undirected graph (in other words, given a graph and an MST $T$, find the smallest among all spanning trees distinct from $T$). Prove that it works.