1. Use Dijkstra’s algorithm to find a shortest path from $a$ to $b$ in the following graph.

![Graph](image)

2. Use the approximation algorithm from class to find a 2-approximation to the minimum-weight Steiner tree in the following graph (the circled vertices are the ones that should be connected by the tree).

![Graph](image)

3. Given a graph $G$ with a matching $M$, define the auxiliary graph $D_M$, show how to use a shortest path in this graph to find an augmenting path or a flower, and prove that this works.

4. Prove that there is no $k$-approximation algorithm for the TRAVELLING SALESMAN PROBLEM for any $k \in \mathbb{R}$, unless $P = NP$.

5. Show that First Fit is a 2-approximation algorithm for BIN PACKING. (Recall that First Fit puts each item in the first bin that it fits into, but it does not sort the items.)

6. Give a polynomial algorithm for finding the shortest cycle in an undirected graph with positive weights, or prove that this is $NP$-hard. Same question for the longest cycle.

7. Let $G$ be an undirected graph without isolated vertices. An edge cover is a subset of the edges such that each vertex is an endpoint of one of these edges.

   (a) Give a polynomial algorithm for finding the minimum cardinality edge cover.

   (b) Suppose the edges have positive weights. Give a polynomial algorithm for finding the minimum weight edge cover.

      (Hint: Find a matching in a graph constructed from two copies of $G$.)