$K_{3,3}$ is not planar

Take a 6-cycle in $K_{3,3}$
$K_{3,3}$ is not planar

Out of three remaining edges two either go inside the cycle,
$K_{3,3}$ is not planar

or two go outside the cycle.
$K_5$ is not planar

Take any 4-cycle in $K_5$. 
$K_5$ is not planar

One of its two diagonals goes inside, the other goes outside.
$K_5$ is not planar

No matter where we put the fifth vertex, it cannot be connected to one of the vertices without crossings.
$K_5$ is not planar

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$K_5$ is not planar

No matter where we put the fifth vertex, it cannot be connected to one of the vertices without crossings.
The Petersen graph
The Petersen graph is not planar, because it contains a subdivision of
Options: $K_5; K_{3,3};$ both $K_5$ and $K_{3,3};$ don’t confuse us, it is planar!
A subdivision of $K_{3,3}$ in the Petersen graph
A subdivision of $K_{3,3}$ in the Petersen graph
A subdivision of $K_{3,3}$ in the Petersen graph
Planar graphs and polytopes

Is it true that the graph of the cube is planar?
Planar graphs and polytopes

Is it true that the graph of any convex polytope in $\mathbb{R}^3$ is planar?
Chromatic number and degrees

Which of the following inequalities hold for any graph with maximum degree $\Delta$ and chromatic number $\chi$?

Options: $\chi \leq \Delta$, $\chi \leq \Delta + 1$, $\chi \geq \Delta$, $\chi \geq \Delta + 1$. 
Which of the following inequalities hold for any graph with minimum degree $\delta$ and chromatic number $\chi$?

Options: $\chi \leq \delta$, $\chi \leq \delta + 1$, $\chi \geq \delta$, $\chi \geq \delta + 1$. 
Take \( v \) of degree 5 and consider its neighborhood. It must be 5-colored.
Five-color theorem

Restrict the attention only to the red and green vertices.
If $x_1$ and $x_3$ are not connected in this restricted graph of red and yellow vertices, then switch red and green in the component of $x_1$. This is a proper coloring.
Color $v$ in red.
There must be a red-green path between $x_1$ and $x_3$. Similarly, there must be a blue-yellow path between $x_2$ and $x_4$. They cannot intersect, but they should.