Petersen graph

What is the girth of this graph?
The girth is equal to 5.
What is the circumference of this graph?
The circumference is equal to 9.
The number of edges in the 10-cycle that cross from the outer 5-cycle to the inner 5-cycle must be even.
All the other vertices on the outer cycle must be connected by the edges from the outer cycle.
Petersen graph. Case 1

We can include in the cycle either the red, or the blue vertices, since they lie on the different sides of the inner 5-cycle with respect to the black vertices.
Petersen graph. Case 2
We can include in the cycle either the red, or the blue vertices, since they lie on the different sides of the outer 5-cycle with respect to the black vertices.
Petersen graph. Case 3
The following is the only way to include the vertices of the outer cycle in the 10-cycle.
Petersen graph. Case 3

Including the blue vertex will produce two disjoint 5-cycles.
Hamiltonian cycles. Necessary condition

The left graph does not satisfy the necessary condition, while the right one does. But both do not contain a Hamiltonian cycle.
When the graph is connected?

What is the maximum number of edges in a graph on \( n \) vertices, if this graph is not connected? Options: \( \binom{n}{2}, \binom{n-1}{2}, \binom{n-1}{2} + 1, n - 1 \).
When the graph is connected?

What is the maximum possible value of the minimum degree in a graph on $2n$ vertices, if this graph is not connected? Options: $2n - 1$, $2n - 2$, $n$, $n - 1$. 
When the graph is connected?

What is the maximum possible value of the maximum degree in a graph on $2n$ vertices, if this graph is not connected? Options: $2n - 1$, $2n - 2$, $n$, $n - 1$. 
Hamilton cycle in the graphs with large minimal degree

Take the longest path $x_1 \ldots x_k$. 
Hamilton cycle in the graphs with large minimal degree

All edges from $x_1$ and $x_k$ go to the vertices of the path.
Hamilton cycle in the graphs with large minimal degree

By pigeon-hole, find $i$ such that $x_{i+1}x_1$ and $x_ix_k$ are both edges.
Hamilton cycle in the graphs with large minimal degree
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This is a Hamiltonian cycle: if not, we could have made a longer path.
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