You can hand in the star problem before 10:15am on Thursday March 24th.

1. Show that in any graph $G$ we have $m(G) \geq \frac{1}{2}vc(G)$.

2. Show that $\alpha(G) + m(G) \leq |V(G)|$.

3. Determine the parameters $\alpha(G), m(G), vc(G), ec(G), dom(G)$ for the Petersen graph $G$.

4. For each of the parameters $\alpha(G), m(G), vc(G), ec(G), dom(G)$, characterize the connected graphs $G$ for which the parameter equals 1.

5. Find a tree $T$ with at least 2 vertices and $dom(T) \neq vc(T)$.

6. (We removed the problem about dominating sets on chessboards, because actually we could not find a nice solution without tedious case analysis.)

7. Show that if $G$ is $k$-regular with $k \geq 1$, then $\alpha(G) \leq \frac{1}{2}|V(G)|$.

8. Show that if $G$ has no isolated vertices, then $m(G) \geq \frac{|V(G)|}{\Delta(G)+1}$.

9. Show that for any graph $G$ we have $dom(G) \geq \frac{1}{3}(diam(G) + 1)$.

*10. Prove that for any graph $G$, it is possible to cover $V(G)$ by vertex-disjoint paths using at most $\alpha(G)$ paths; in other words, there exist paths $P_1, \ldots, P_k$ with $k \leq \alpha(G)$ such that $V(P_i) \cap V(P_j) = \emptyset$ for all $i \neq j$, and $\bigcup V(P_i) = V(G)$.