1. For some $n$, give a graph with $n$ vertices, $n + 3$ edges, and exactly 8 cycles.

2. Find two non-isomorphic graphs with the same number of vertices and the same sequence of degrees.

3. Show that any graph with at least two vertices has two vertices with the same degree.

4. What is the maximum number of edges in a bipartite graph on $n$ vertices? Prove it.

5. Let $G$ be a graph in which every vertex has even degree. Show that $E(G)$ can be partitioned into the edge sets of cycles.

6. A graph is $k$-regular if every vertex has degree $k$. Describe all 1-regular graphs and all 2-regular graphs. Show that a graph on an odd number of vertices cannot be 3-regular, while for every even $n \geq 4$, there is a 3-regular graph on $n$ vertices.

7. Show that any graph with at least 6 vertices contains 3 vertices that are pairwise adjacent, or 3 vertices that are pairwise non-adjacent.

8. Let $G$ be a graph containing a cycle $C$, and assume that $G$ contains a path of length at least $k$ between two vertices of $C$. Show that $G$ contains a cycle of length at least $\sqrt{k}$.

*9. Prove the statement of Problem 8 with $\sqrt{2k}$ instead of $\sqrt{k}$.

*10. Show that every connected graph $G$ contains a path of length at least $\min\{2\delta(G), |G| - 1\}$.