1. Show that a graph is bipartite if and only if it has no odd cycles.

2. Suppose that $G$ is a graph with $|V(G)| \geq 3$ such that for all $x \in V(G)$, $G - x$ is a tree. Determine what kind of graph $G$ is, and prove it.

3. Show that a matching $M \subset E(G)$ is a maximum matching if and only if there is no augmenting path for $M$.

4. Show that if every two odd cycles of $G$ intersect in at least one vertex, then $\chi(G) \leq 5$.

5. Let $G$ be a graph containing a cycle. Prove that $\alpha(G) \geq \left\lfloor \frac{1}{2} \text{gir}(G) \right\rfloor$.

6. Prove that a graph is 2-connected if and only if for every three vertices $x, y, z$, there is a path from $x$ to $z$ that passes through $y$.

7. State and prove Euler’s formula for connected planar graphs.

8. Prove that if $G$ is a graph with $|V(G)| \geq 4$ and $|E(G)| \geq 2|V(G)| - 3$, then $G$ contains a cycle with a chord (a chord of a cycle is an edge that is not part of the cycle, but that connects two vertices from the cycle).

9. Prove that a graph with $n$ vertices and $e$ edges has at least $\frac{e}{n} \left( e - \frac{1}{4}n^2 \right)$ triangles.