1. Is the following statement true or false? \( n! = O(n^n) \).

2. Is the following statement true or false? A graph is a tree if and only if it has no cycle.

3. Is the following statement true or false? Adding an edge between two non-adjacent vertices of a tree always creates precisely one cycle.

4. Is the following statement true or false? The number of trees on 10 vertices is \( 8^{10} \).

5. Is the following statement true or false? Every non-empty finite set has as many subsets with an odd number of elements as many with an even number of elements.

6. Is the following statement true or false? For every partially ordered set \((X, \leq)\) and every positive integer \(k\), there exists a covering of \(X\) by \(k\) chains if and only if \(X\) contains no \(k + 1\) pairwise incomparable elements.

7. Let \(k\) be a positive integer, and let \(A_1, A_2, \ldots, A_m\) be different \(3k\)-element subsets of a \(4k\)-element set such that no two of them are disjoint. Is the following statement true or false? \(m \leq \binom{4k-1}{3k-1}\).

8. Let \(F_1, F_2, F_3, \ldots\) denote the Fibonacci numbers. Is the following statement true or false? For every positive integer \(n\), we have \(F_1 + F_2 + \ldots + F_n = F_{n+2} - 1\).

9. Let \(k\) and \(n\) be positive integers such that \(k \leq n\). Write down the formulas for
   - the number of all sequences \((a_1, a_2, \ldots, a_k)\) with \(a_1, a_2, \ldots, a_k \in \{1, 2, \ldots, n\}\);
   - the number of sequences \((a_1, a_2, \ldots, a_k)\) such that \(a_1, a_2, \ldots, a_k \in \{1, 2, \ldots, n\}\) and \(a_1, a_2, \ldots, a_k\) are pairwise distinct;
   - the number of sequences \((a_1, a_2, \ldots, a_k)\) such that \(a_1, a_2, \ldots, a_k \in \{1, 2, \ldots, n\}\) and \(a_1 < a_2 < \ldots < a_k\).

10. An infinite sequence \((a_n)_{n=0}^\infty\) is defined by the following recurrence equation:
    \[
    a_0 = 1, \quad a_1 = 2, \quad a_{n+2} = 2a_{n+1} - a_n.
    \]
    Write down the generating function of the sequence \((a_n)_{n=0}^\infty\). Provide the formula for \(a_n\), and explain how this formula follows from the generating function of \((a_n)_{n=0}^\infty\).

11. Let \(G\) be a graph with \(m\) edges, and let \(k\) be a positive integer. Prove that the vertices of \(G\) can be colored with \(k\) colors so that at most \(m/k\) edges connect two vertices with the same color.

12. Recall that a vertex of a tree is called a leaf if it has degree 1. Prove that every tree with \(n\) vertices (where \(n \geq 2\)) and with no vertex of degree 2 has at least \((n+2)/2\) leaves.

13. Let \(n\) be a positive integer, and let \(\mathcal{F}\) be a family of subsets of the set \(\{1, 2, \ldots, n\}\) with the property that any two sets in \(\mathcal{F}\) have non-empty intersection. Prove that \(|\mathcal{F}| \leq 2^{n-1}\).