1. How many ways are there to place \( n \) undistinguishable rooks on the \( n \times n \) chessboard so that no two of the rooks attack each other? (Rook is a chess piece that moves horizontally or vertically through any number of unoccupied squares.)

2. Given a number \( n \in \mathbb{N} \), compute the number of subsets of the set \{1, 2, \ldots, n\} which have
   - odd size,
   - even size.

Hint: Prove that the number of subsets of odd size is equal to the number of subsets of even size.

3. Provide combinatorial proofs of the following identities:

   \[
   \binom{n}{k} = \binom{n}{n-k},
   \sum_{k=0}^{n} \binom{a}{k} \binom{b}{n-k} = \binom{a+b}{n},
   \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = (a+b)^n.
   \]

4. Given numbers \( n, k \in \mathbb{N} \), count the number of sequences \((x_1, x_2, \ldots, x_k)\) of natural numbers (including zero) such that \( x_1 + x_2 + \ldots + x_k = n \).

5. Given numbers \( n, k \in \mathbb{N} \), count the number of sequences \((x_1, x_2, \ldots, x_k)\) of numbers from the set \{1, 2, \ldots, n\} such that \( x_1 \leq x_2 \leq \ldots \leq x_k \).

Hint: Count the sequences \((x_1, x_2 + 1, \ldots, x_k + k - 1)\).

6. Given numbers \( n, k \in \mathbb{N} \), count the number of sequences \((X_1, X_2, \ldots, X_k)\) of sets such that \( X_1 \subseteq X_2 \subseteq \ldots \subseteq X_k \subseteq \{1, 2, \ldots, n\} \).

7. * A pawn lies on the lower left square of the \( n \times n \) chessboard. We repeatedly move the pawn by one square up or one square right until it reaches the upper right square. How many different paths can we choose for the pawn through the chessboard?