1. Prove using induction that a planar graph with \( n \geq 3 \) vertices has at most \( 3n - 6 \) edges.

2. Show that the bound on \( cr(G) \) in the crossing lemma is optimal up to a constant factor.
   (Hint: Take the disjoint union of complete graphs as \( G \).)

3. Show that the bound on the number of point-line incidences in Szemerédi-Trotter theorem is optimal up to a constant factor. To this end, for \( r, s \in \mathbb{N} \), find a configuration of \( 2r^2s \) points and \( rs^2 \) lines in the plane such that the number of incidences between these points and lines is equal to \( r^2s^2 \).
   (Hint: Take the integer grid \( \{1, \ldots, r\} \times \{1, \ldots, 2rs\} \) as the set of points.)

4. Prove that there is an absolute constant \( c > 0 \) with the following property: for any set \( X \) of \( n \) points in the plane, the number of lines passing through at least \( k \geq 2 \) points in \( X \) is at most \( c(n^2/k^3 + n/k) \).
   (Hint: Use Szemerédi-Trotter theorem.)

5. In a manner similar to the proof of Szemerédi-Trotter theorem, prove that there is an absolute constant \( c > 0 \) such that the number of incidences between \( n \) points and \( m \) unit circles in the plane is at most \( c(m^{2/3}n^{2/3} + m + n) \). Be careful in handling possible multiple edges in the graph considered in the proof.

6. Prove that there is an absolute constant \( c > 0 \) with the following property: for any set \( X \) of \( n \) points in the plane, the number of pairs of points in \( X \) at distance exactly 1 is at most \( cn^4/3 \).
   (Hint: Use the result of problem 5.)
   (Remark: Compare with problem 4 in Problem Set 3.)

7. * Let \( L \) be a set of lines in the plane and \( p \) be a point not belonging to any of these lines. We say that lines \( \ell_1, \ell_2 \in L \) intersect at level \( k \) with respect to the point \( p \) if the segment between \( p \) and the intersection point of \( \ell_1 \) and \( \ell_2 \) crosses \( k \) lines in \( L \).
   Prove that there is an absolute constant \( c > 0 \) with the following property: for any set \( L \) of \( n \) lines in the plane, any point \( p \) not belonging to any of these lines, and any \( k \in \mathbb{N} \), the number of intersections of lines in \( L \) at level \( k \) with respect to \( p \) is at most \( c(k + 1)n \).
   (Hint: Use the idea of random amplification, like in the proof of the crossing lemma: take a random subset of the lines, and apply a trivial bound on the number of intersections at level 0 for this subset.)