Advanced Discrete Math – Practice Exam – EPFL 2013

This practice exam should give you an idea of what the real exam will look like. The real exam should not be more difficult, but may be less difficult. The fact that a question is on the practice exam will not affect the probability that it occurs on the real exam.

Practice Exam

1. Show that if a set system $(X, \mathcal{S})$ satisfies $|S| \bmod 11 \notin \{1, 4, 7\}$ for all $S \in \mathcal{S}$ and $|S \cap T| \bmod 11 \in \{1, 4, 7\}$ for all $S, T \in \mathcal{S}$, then

$$|\mathcal{S}| \leq \sum_{i=0}^{3} \binom{|X|}{i}.$$

2. Prove that if G is a graph with $e \geq 4n$, then its crossing number satisfies

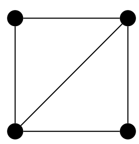
$$\mathrm{cr}(G) \geq \frac{e^3}{64n^2}.$$

3. Show that if G is d-regular, then the multiplicity of its largest eigenvalue λ_1 equals the number of its connected components.

4. Prove that for every $\alpha > 0$ there is $n_0 \in \mathbb{N}$ such that every graph G with $n \geq n_0$ vertices and at least

$$\left(\frac{1}{4} + \alpha\right) n^2$$

edges contains a copy of



Note: You may not use the statement of Erdős-Stone, but you can use its proof, or any other statement from the course.

5. Show that if G has no isolated vertices, then there are two disjoint independent sets $A, B \subset V(G)$ such that

$$|A| + |B| \geq \sum_{v \in V(G)} \frac{2}{d(v) + 1}.$$

6. We call a graph G *awesome* if there are numbers k, l, m with $m > 0$ such that

   - G is k-regular;
   - Any two neighbors have l common neighbors;
   - Any two non-neighbors have m common neighbors.

   For instance, C_5 and the Petersen graph are awesome. Prove that a regular connected graph is awesome if and only if it has exactly three distinct eigenvalues.
