Exercises Week 9

1 Exercises

1. In case you have been feeling lazy, do prove last weeks bonus exercise: Prove by induction on the number of faces that a plane graph $G$ is bipartite if and only if every face has even length (if both sides of an edge are in the same face then it is counted two times in the length of the face).

2. We have seen in class that any 3-connected graph without a $IK_{3,3}$ and $IK_5$ minor is planar. Using the ideas from that proof, show that you can draw a 3-connected planar graph in such a way that every edge is straight and every face is convex.

3. Construct a planar graph that cannot be drawn with all faces convex.

4. A graph is called outerplanar if it has a drawing in which every vertex lies on the boundary of the outer face. Show that a graph is outerplanar if and only if it contains neither $IK_4$ nor $IK_{2,3}$ as a minor.

5. Does every minimal non-planar graph $G$ (i.e., every non-planar graph $G$ whose proper subgraphs are all planar) contain an edge $e$ such that $G - e$ is maximally planar? Does the answer change if we define minimal with respect to minors rather than subgraphs?

Bonus Problem: Show that every planar graph is a union of three forests. Hint: You may use Theorem 2.4.4.

Given out: Thursday, April 23