Exercises Week 6

1. Let $e$ be an edge in a 2-connected graph $G \neq K_3$. Show that either $G - e$ or $G/e$ is 2-connected.

2. What is the minimum number of crossings necessary to draw $K_{3,3}$ and $K_5$ in the plane?

3. Let $G$ be a $d$-regular graph with $n$ vertices. How many edges does $L(G)$ have?

4. Show that every graph can be embedded in $\mathbb{R}^3$ with straight-line edges.

5. Let $G$ be a bipartite planar graph such that the size of each of its faces is at least 4. Show that $G$ has at most $2n - 4$ edges.

**Bonus Problem:** A football is made of pentagons and hexagons, not necessarily of regular shape. They are sewn together so that their seams form a 3-regular graph. How many pentagons does the football have?