Exercises Week 5

1 Understanding the definitions

1. In the following network find a maximum flow from $s$ to $t$ and a minimum $st$-cut.

2. Let $K_n$ be the complete graph on $n$ vertices and assume all of the edges have a capacity of one. What is the maximum flow between any two vertices?

3. Let $C_n$ be a cycle with $n$ vertices and assume all of the edges have a capacity of one. What is the maximum flow between any two vertices?

4. Calculate the vertex and edge connectivity of the following graph. Thereafter, chose a vertex and delete it. Calculate the new edge and vertex connectivity. Does the choice of vertex matter?
2 Exercises

1. Using the proposition about 2-connected graphs seen in class (Prop 3.1.1 in the book) prove the following statement: a graph is 2-connected graph if and only if for every pair of vertices there are two pairwise disjoint paths between them.

2. Let $G$ be a $k$-connected graph, and let $xy$ be an edge of $G$. Show that $G/xy$ is $k$-connected if and only if $G - \{x, y\}$ is $(k - 1)$-connected.

3. Let $G$ be a graph and $a, b$ be two vertices of $G$. Let $X \subseteq V(G) \setminus \{a, b\}$ be an $a - b$ separator in $G$. Show that $X$ is minimal by inclusion (remember that this is different than minimum) as an $a - b$ separator if and only if every vertex in $X$ has a neighbour in the component $C_a$ of $G - X$ containing $a$, and another in the component $C_b$ of $G - X$ containing $b$.

4. Let $e$ be an edge in a 2-connected graph $G \neq K_3$. Show that either $G - e$ or $G/e$ is 2-connected.

**Bonus Problem:** Prove Hall’s theorem using Menger’s theorem.