Exercises Week 4

1 Understanding the definitions

For the following practice exercises let \( G = (V, E) \) be a graph and \( A \) and \( B \) be two subsets of \( V \).

1. Assume \( |A| = |B| = 2 \), what is the maximum possible number of \( A - B \) disjoint paths in \( G \).

2. Let \( X \) be a subset of \( V \) such that \( X \) separates \( A \) and \( B \). Assume \( |A| = |B| = |X| = k \) and that there is a set of \( k \) disjoint \( A - X \) paths and another set of \( k \) disjoint \( X - B \) paths. Can you guarantee the existence of \( k \) disjoint \( A - B \) paths?

3. Let \( K_n \) denote the complete graph on \( n \) vertices (defined in Assignment 2). Draw the line graph of \( K_3, K_4 \) and \( K_5 \).

4. Find a graph \( G \) such that the line graph of \( G \) is \( K_n \).

5. What is the vertex connectivity of \( K_3 \) and \( K_4 \)? What is the edge connectivity of \( K_3 \) and \( K_4 \)?

2 Exercises

1. Find an algorithm that finds a matching of maximum size, for every graph \( G = (V, E) \). Your algorithm should run in at most \( c|V||E| \) steps, for some (absolute) constant \( c \) that does not depend on the size of the graph.

2. Let \( G \) be a graph on \( 2n \) vertices, such that all degrees are at least \( n \). Show that \( G \) has a perfect matching.

3. Show that a partially ordered set of at least \( rs + 1 \) elements contains either a chain of size \( r + 1 \) or an antichain of size \( s + 1 \).

Given out: Thursday, March 12
**Bonus Problem:** Let $P = \{v_1, \ldots, v_{n+1}, v_{n+2}\}$ be a set of $n$ points in the plane. For $i, j > 2$ we say that $v_i \prec v_j$ if the triangle $v_j v_1 v_2$ contains $v_i$ in its interior. If neither $v_i \prec v_j$ nor $v_j \prec v_i$ is true, then we say $v_i \nleq v_j$. Show that there exists a subset $Q = \{y_1, \ldots, y_{\lfloor \sqrt{n} \rfloor}\} \subseteq P$ of size at least $\sqrt{n}$ such that either $p \nleq q$ for all $p, q \in Q$ or $y_i \prec y_j$ for all $i < j$. 

*Given out: Thursday, March 12*