Exercises Week 11

**Definition.** For graphs $H_1$ and $H_2$ we define $R(H_1, H_2)$ to be the smallest integer $n$ such that for every graph $G$ with $n$ vertices either $H_1$ is an induced subgraph of $G$ or $H_2$ is an induce graph of $G$.

1. Show that $R(K_3, K_3) = 6$.

2. Calculate $R(P_3, K_3)$ and $R(P_4, P_4)$ ($P_k$ is a path with $k$ vertices).

3. Show that $R(K_s, K_t) \leq R(K_{s-1}, K_t) + R(K_s, K_{t-1})$

4. For a reminder of some basic probability:
   a. Imagine we throw a coin 100 times, what is the expected number of tails.
   b. Imagine we throw a fair dice 5 times. Let $X$ be the sum of the 5 obtained values. What is bigger $P(X \leq 5)$ or $P(X \geq 30)$.
   c. Imagine we throw a fair dice 2 times. Let $X$ be the sum of the two obtained values. Calculate $P(X$ is odd).
   d. A normal deck consists of 52 cards, 13 of each suit, 2 red suits and 2 black suits. What is the probability of taking two cards at random and obtaining a pair? And what is the probability of taking two cards at random and having a pair of cards with the same colour?

**Bonus Problem:** For every $s \in \mathbb{N}$ there exists a constant $c$ such that every graph on $n$ vertices with no $K_{s,2}$ graph has at most $cn^{3/2}$ edges.