Exercises Week 1: *Introduction*

1. (a) Is there a graph with degree sequence: 3, 3, 3, 5, 6, 6, 6, 6, 6, 6?  
   (b) Is there a graph with degree sequence: 1, 1, 3, 3, 3, 5, 6, 8, 9?

2. Show that every graph $G$ has a path of length $\delta(G)$ and a cycle of length $\delta(G) + 1$.

3. Let $T$ be a tree: that is a connected graph with no cycles. Show that if we add any other edge to $T$ then a cycle must be created, and that if we remove any edge from $T$ then the graph becomes disconnected.

4. Let $T$ be a tree. Show that the number of leaves is at least $\Delta(T)$.

5. Let $G$ be an eulerian graph and $e \in E(G)$. Show that if we remove $e$ from $G$ then $G$ will still be connected.

6. We say that a graph has an eulerian path if there is a path that passes through each edge exactly once. Show that a graph has an eulerian path if and only if there are at most two odd degree vertices.

**Bonus Problem:** Among the people taking a course of graph theory we know that no student feels lonely during class: that is they have at least one friend in the class. Show that there are two people in the class that have exactly the same amount of friends in the class.

- The assignment is due on Thursday, February 20 at the exercise session
- Submit a solution to the bonus problem *only*